OPERATIONAL RESEARCH
(TRANSPORTATION MODEL)

Introduction-Transportation Problems

- Transportation problems is a type of Linear Programming Problem
- The total cost of transportation is minimized.
- In transportation problem, the variables are integers.

Mathematical Formulation of Transportation Problem

- Let there be three units, producing Veggies say, A_1, A_2 and A_3 from where the Veggies are to be supplied to four street markets say B_1, B_2, B_3 and B_4.
- Let the number of veggies produced at A_1, A_2 and A_3 be a_1, a_2 and a_3 respectively and the demands at the Street Market be b_1, b_2, b_3 and b_4 respectively.
- We assume the condition
  - $a_1 + a_2 + a_3 = b_1 + b_2 + b_3 + b_4$

- Let the cost of transportation of one Veggies from A_1 to B_1 be c_{11}. Similarly, the cost of transportations in other cases are also shown in Table 1.
- Let out of a_1 veggies available at A_1, x_{11} be taken at B_1 street market, x_{12} be taken at B_2 street market and to other Street market as well, as shown in table 1.
- Total number of Veggies to be transported form A_1 to all destination, i.e., B_1, B_2, B_3, and B_4 must be equal to a_1.
  - $x_{11}+x_{12}+x_{13}+x_{14} = a_1$ -------------------------(1)

- Similarly, from A_2 and A_3 the veggies transported be equal to a_2 and a_3 respectively.
  - $x_{21}+x_{22}+x_{23}+x_{24} = a_2$ ---------------------(2)
  - $x_{31}+x_{32}+x_{33}+x_{34} = a_3$ ---------------------(3)
- On the contrary
- $x_{11}+x_{21}+x_{31} = b_1$ ----------------------------(4)
- Similarly,
  - $x_{12}+x_{22}+x_{32} = b_2$ -------------------------(5)
  - $x_{13}+x_{23}+x_{33} = b_3$ -------------------------(6)
  - $x_{14}+x_{24}+x_{34} = b_4$ -------------------------(7)

- With the help of the above information we can construct the following table :

Unit \ Depot	To B_1	To B_2	To B_3	To B_4	Stock
From A_1	$x_{11}(c_{11})$	$x_{12}(c_{12})$	$x_{13}(c_{13})$	$x_{14}(c_{14})$	a_1
From A_2	$x_{21}(c_{21})$	$x_{22}(c_{22})$	$x_{23}(c_{23})$	$x_{24}(c_{24})$	a_2
From A_3	$x_{31}(c_{31})$	$x_{32}(c_{32})$	$x_{33}(c_{33})$	$x_{34}(c_{34})$	a_3
Requirement	b_1	b_2	b_3	b_4	

Balanced Transportation Problem

c_{11}=2	c_{12}=3	c_{13}=5	c_{14}=1	a_1=8
c_{21}=7	c_{22}=3	c_{23}=4	c_{24}=6	a_2=10
c_{31}=4	c_{32}=1	c_{33}=7	c_{34}=2	a_3=20
b_1=6	b_2=8	b_3=9	b_4=15	= =38

- Methods to solve transportation problem.
  - North-West corner rule
  - Lowest cost entry method
  - Vogel's approximation method

Note

- After calculating the cost of transportation by the above three methods, one thing is clear that Vogel's approximation method gives an initial basic feasible solution which is much closer to the optimal solution than the other two methods. It is always worth while to spend some time finding a "good" initial solution because it can considerably reduce the total number of iterations required to reach an optimal solution.

Set an Example

- For the transportation problem, find an initial basic feasible solution by the three methods described.

Factory \ Warehouse →	W_1	W_2	W_3	W_4	Factory Capacity
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Warehouse Requirement	5	8	7	14	34

Unbalanced Transportation Problem

- Total supply ≠ Total demand

$$\sum_{i=1}^{m} a_i \neq \sum_{i=1}^{n} b_i$$

Unbalanced Transportation Problem

- In real-life, supply and demand requirements will rarely be equal.

- Demand variations may fluctuate due to several factors(weather, inflations)

Unbalanced Transportation Problem

- How to solve UTP?

dummy sources or dummy destinations.

- **Demand Less than Supply**
  - A dummy destination (dummy column) with demand equal to the supply surplus is added.

- **Demand Greater than Supply**
  - A dummy source (dummy row) with supply equal to the demand surplus is added.

The unit transportation cost for the dummy column and row are assigned zero values.

Example 9

- Find the initial transportation cost for the problem using the North-West Corner method.

	1	2	3	4	5	Supply
A	4	13	12	5	9	7
B	3	6	9	4	3	11
C	5	11	6	10	14	12
Demand	4	6	6	7	9	

Case Study 1

- The values in the table shows the profit obtained (in Rs) for an item sold in four districts. The demand for each district is given. There are three supply points which has to satisfy the demand of all four districts. Use the Vogel's method to obtain an initial basic feasible solution. The objective is to maximise profit.

		Districts				
		D_1	D_2	D_3	D_4	Supply
Supply Point	1	61	71	81	11	5
	2	32	28	74	62	10
	3	53	44	22	61	15
	Demand	7	8	5	10	

Maximization Transportation Problem

Note : If you have a maximation problem you need to convert it.

How ?

By subtracting all the units cost from highest unit cost given in the table and solve.